

# Engineering Notes

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## Limit Cycle Oscillation Characteristics of Fighter Aircraft

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### Introduction

**L**IMIT cycle oscillations (LCO) have been a persistent problem on several fighter aircraft.<sup>1</sup> The F-16 and F/A-18 encounter LCO at high subsonic and transonic speeds for store configurations with AIM-9 missiles on the wingtips and heavy stores on the outboard pylons. The LCO response is characterized by antisymmetric motion of the wing and stores and a lateral motion of the fuselage and aircrew. This limit cycle behavior occurs in both level flight and during elevated aircraft load factor maneuvers. It may be self-induced or initiated by control inputs. Once started, the oscillations are self-sustaining and persist until flight conditions (e.g., airspeed, altitude, and aircraft load factor) are suitably altered.

This phenomenon is considered to be closely linked to classical flutter, except that the coupling of the structural response and the unsteady aerodynamic forces is nonlinear in nature, resulting in a limited amplitude oscillatory motion.<sup>2</sup> Because of this close relationship to flutter and the apparent noncatastrophic nature of LCO, much confusion exists concerning the best way to regard this phenomenon. This Note presents some observations on LCO and discusses the evolution of the terminology associated with this phenomenon. Some observed distinctions and similarities between LCO and classical flutter, descriptions of the aircraft response characteristics, and impacts of LCO on test procedures and other areas are also discussed.

### Terminology

LCO is a term that has come into widespread use since the mid-1970s to describe the aeroelastic response of certain aircraft and external store configurations that encounter sustained, periodic, but not catastrophically divergent motion in portions of the flight envelope. Figure 1 shows an example of LCO as it would appear on a strip-chart recorder. Other terms that have been used to describe this type of behavior in the past include limit cycle flutter and limited amplitude flutter.

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The use of the word flutter to describe this behavior has been challenged by aircraft manufacturers, their corresponding governmental sponsors, and those in the flutter community who felt it unfair to associate a word that evokes visions of sudden catastrophic structural failures with a phenomenon that does not normally exhibit such behavior. In addition, there are engineers who do not believe that flutter is the root cause of LCO. For instance, some believe that the genesis of LCO is in the flight control system, whereas others argue that the behavior cannot be flutter because classical flutter analysis techniques are unable to predict the limited amplitude nature of the phenomenon.

Those who prefer to retain the word flutter in the descriptor also have several valid points. To begin with, the phenomenon is normally first brought to light during flutter flight testing, and flight testing is accomplished by a flutter flight test team using typical flutter flight test procedures. In addition, many believe the root cause of the phenomenon is indeed in flutter. The strongest argument in favor of this position is that the inflight behavior satisfies all aspects except one (the catastrophically divergent oscillations at speeds greater than the onset speed) of classical flutter. Once LCO is well established, the external stores and all parts of the aircraft vibrate in a single mode, at a single frequency. Another supporting argument is that flutter analyses do an excellent job of predicting the frequency of the LCO, and the predicted flutter speed based on a damping value of zero (as opposed to the commonly used 2% damping value) is frequently quite close to the LCO onset speed in straight-and-level flight.<sup>3</sup>

The term LCO ignores the origin issue and simply describes the nature of the motion associated with the phenomenon. Specifically, the amplitude of the motion is limited (constant for stabilized flight conditions), the motion is cyclic (repetitive in a given period of time), and it is oscillatory (the amplitude varies above and below a mean value). What this means is that LCO in its purest form is characterized by sinusoidal motion.

For typical LCO the amplitude is constant only in stabilized flight conditions. Once above the onset speed and accelerating to a new higher speed, the amplitude of the motion grows and appears to be diverging until the new target speed is reached. When speed is again stabilized, the motion will again become sinusoidal, but with a bigger amplitude than it had at the earlier speed.

### Aircraft Response Characteristics

For most aircraft that experience LCO, the motion is dominated by antisymmetric modes and is felt by the aircrew as a lateral motion. Reference 3 presents a possible explanation as to why LCO seems to be predominantly an antisymmetric vs symmetric phenomenon.

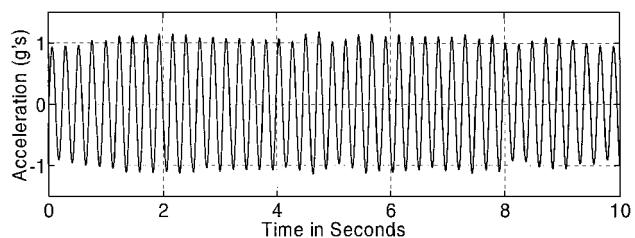


Fig. 1 Example of LCO.

The hypothesis centers around the dominance of outer vs inner wing deflections and the relative ease with which antisymmetric motions can transfer energy across the fuselage to the opposite wing. Depending on the amplitude and frequency involved, the LCO-induced motion of the crew may result in an inability to read cockpit displays or accomplish flight or mission related tasks such as pushing buttons or flipping switches that require precise, correct-the-first-time manual performance.

LCO occurs in the high subsonic to low supersonic speed regime, and has most often manifested itself on aircraft with wingtip missile launchers, such as the F-5, F-16, and F/A-18 (Ref. 1). LCO may be self-induced or initiated by control inputs. Once started, the oscillations are self-sustaining and persist until flight conditions are suitably altered. Note that when backing out of an LCO condition, the oscillations frequently persist to airspeeds or aircraft load factors lower than the original onset values.

Limit cycle behavior occurs both in level flight and during elevated aircraft load factor maneuvers. It is generally believed that the occurrence of LCO during turns and pull-ups is driven more by angle-of-attack related changes on the wingtip flowfield associated with these maneuvers than by the inertial effects. It is common for LCO onset speeds to be lower in elevated load factor maneuvers than in straight-and-level flight. The LCO behavior during windup turns vs that during straight-and-level accelerations is much less predictable. As an example, amplitudes have been known to grow as expected when going from 1 to 4 g and then diminish as load factor continues to be increased until there is no LCO evident at 5.5 g. For other store configurations on the same aircraft, amplitudes steadily increase with increasing load factor throughout the test event. On still other configurations, there is no discernible LCO activity until approximately 5 g, at which time the LCO begins and grows so rapidly that it has all of the appearances of diverging in the manner of classical flutter.

There are also instances in which the aeroelastic response is characterized by a slow but steady increase in the amplitude of the oscillations while flight conditions are held constant. Persistent growth in amplitude after the flight conditions are stabilized is not characteristic of LCO. This type of behavior is, however, characteristic of what one would expect to see during classical flutter involving a critical mode whose damping is only slightly negative. Even in light of this characteristic, the term LCO is still often used to describe this aeroelastic instability. The term divergent LCO has even been used in some instances despite the obvious contradiction in terminology.

### Impact on Testing Procedures

As discussed earlier, LCO has presented something of a dilemma to the flutter engineering community with regard to semantics; however, it creates an even greater quandary with regard to flight-test procedures. Military specifications<sup>4</sup> have long emphasized 3% total damping as the minimum value acceptable within the limit speed/Mach envelope of military airplanes. During flutter flight tests of aircraft not known to exhibit LCO, damping values of 2% or less are definite cause for caution and values of 1% are normally cause for termination of testing. This concern with extremely low values of damping is based on the assumption that classical flutter is imminent, and that any further increase in airspeed may put the aircraft beyond the flutter speed, in which case one or more of the aerodynamic surfaces of the aircraft could experience divergent oscillatory motion resulting in a structural failure.

As mentioned earlier, LCO in its purest form is characterized by sinusoidal motion. From a logarithmic decrement perspective, the damping value associated with sinusoidal motion is zero, hence, the damping value for each succeeding test condition is the same, that is, zero. The customary plot of damping vs airspeed is obviously of no value under these conditions, and, as a result, there is no single, commonly accepted engineering criteria for terminating LCO testing. The U.S. Air Force has adopted a policy for the F-16 based on wingtip accelerometer response,<sup>5</sup> whereas the U.S. Navy's policy for the F/A-18 is based on lateral acceleration levels at the pilot seat.<sup>1</sup> The different criteria result from the analog vs digital

flight control systems each was equipped with in its early stages of development.

In the case of the F/A-18, the degraded cockpit environment was considered to be the main drawback of LCO. With a digital flight control system, simple changes to the software provided a ride quality enhancement system to keep the cockpit environment within acceptable bounds. The system uses the production fuselage lateral accelerometer package as sensors, and cockpit environment suitability is measured in terms of lateral acceleration at the pilot's seat, hence, the Navy's termination criterion is based on lateral acceleration at the pilot's seat.

The F-16, on the other hand, began life with an analog flight control system. Hence, the addition of a ride quality enhancement system could not be achieved with a simple change in software, and placards were imposed for external stores that encountered excessive LCO. Because the testing was being performed by flutter engineers whose focus was already on the wingtip behavior, the test termination policy for the F-16 was couched in terms of wingtip response.

As a final note, there has always been a question as to whether or not it is possible to get so deeply into an LCO condition that the behavior would revert to that of classical flutter. Although no instance of a structural failure has ever been reported, there have been occasions when LCO has suddenly and rapidly grown in amplitude, and it was unclear at what level the divergent behavior would have stopped, or if it would have stopped at all short of a structural failure.

### Other Impacts

In addition to the question of how much LCO is allowable in a test environment, concerns over other possible effects of LCO have been expressed. In the area of human factors, considerations have been raised with regard to the ability of the aircrew to perform combat-related tasks while in an LCO condition and with regard to the possibility that a surprise exposure to LCO would persuade the pilot that something was wrong with the aircraft and result in termination of the mission or a decision to avoid a part of the flight envelope crucial to combat survivability.

Another issue that has received attention in the past is that of fatigue. Because of the low load levels (compared to limit loads) experienced on various parts of the aircraft, fatigue of the basic aircraft structure for normal operational aircraft has been judged not to be a problem. Fatigue concerns for test aircraft that are repeatedly exposed to LCO, however, are still evident. These concerns have been lent credence by the occurrence of fatigue-related failures of components on flutter test aircraft. Although the failures were not attributed to long-term LCO exposure, they are believed to have been caused by long-term exposure to the flight-test environment and repeated exposure to high load cycles produced by the onboard flutter excitation system. Inasmuch as the load cycles experienced during LCO are very similar to those induced by the onboard flutter excitation system, the implications regarding fatigue effects are obvious.

Questions have also been posed concerning possible effects of LCO on ordnance. These issues include such things as possible degradation in reliability of components, whether or not ordnance can be safely released during LCO, the possible effects of LCO on target acquisition for smart weapons, and the effects of LCO on weapon accuracy for unguided weapons.

### Conclusions

In this Note we have attempted to shed some light on the basic nature of LCO and the evolution of the terminology. We also presented some observed distinctions and similarities between LCO and classical flutter and discussed the impacts of LCO on test procedures, aircraft store certification, and mission capability. It should be obvious from the material presented that both further research into the root cause of the phenomenon and improved theoretical prediction tools are needed. Finally, its history of becoming divergent in the manner of classical flutter for certain store combinations clearly demonstrates that appropriate LCO test termination criteria

are necessary and that LCO testing should continue to be performed by engineers well versed in classical flutter flight test procedures.

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## Results of Theodorsen and Garrick Revisited

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### Nomenclature

<i>a</i>	= distance, in semichords, between airfoil midchord and elastic axis (see Fig. 1)
<i>b</i>	= airfoil semichord
<i>h</i>	= airfoil plunge displacement (see Fig. 1)
<i>k</i>	= reduced frequency, $\omega b / V$
<i>m</i>	= airfoil mass (per unit span)
<i>r</i> <sub><i>α</i></sub>	= radius of gyration, in semichords, of airfoil with respect to the elastic axis
<i>V</i>	= airspeed
<i>x</i> <sub><i>α</i></sub>	= distance, in semichords, from airfoil elastic axis to center of mass (see Fig. 1)
<i>α</i>	= airfoil pitch displacement (see Fig. 1)
<i>μ</i>	= airfoil mass ratio, $m / \rho \pi b^2$
<i>ρ</i>	= air mass density
<i>ω</i> <sub><i>h</i></sub>	= uncoupled plunge radian frequency
<i>ω</i> <sub><i>α</i></sub>	= uncoupled pitch radian frequency

### Introduction

DURING the first half of the 20th century, Theodore Theodorsen formulated the first analytically exact unsteady aerodynamic theory for modeling the mechanism of aeroelastic flutter.<sup>1</sup> The case considered was that of the two-dimensional airfoil section, with degrees of freedom in plunge, pitch, and trailing-edge control surface rotation, in unsteady, incompressible flow. Theodorsen with I. E. Garrick, authored several NACA reports<sup>2,3</sup> containing plots of a critical flutter speed parameter for ranges of a variety of airfoil and flow parameters.

The airfoil flutter theory and results of Theodorsen and Garrick<sup>2,3</sup> are likely no longer used by anyone for designing safe, operational vehicles, but they do serve useful purposes. The Theodorsen theory<sup>1</sup> is still a useful educational tool in universities, being the simplest flutter problem that students can prepare computer solutions for with relative ease and at the same time learn the essential character of solving flutter equations. In addition, the Theodorsen flutter

solution, being for two-dimensional, incompressible, inviscid flow, provides a limiting case for any newly developed computational fluid dynamics schemes. Although the theory is flawless, the computational resources available at the time (when computer was a job title!) leave much to be desired when compared to the resources available today. Some years ago, while doing his doctoral work, the present author found<sup>4</sup> a number of erroneous plots in the reports of Theodorsen and Garrick<sup>2,3</sup> and in other work that references their results.<sup>5,6</sup> The amount of heartburn and time that the author spent checking and rechecking could have been saved had it been known that some (if not many, or all!) of the flutter boundaries in the old NACA reports and texts were in error. The same could be said of other research situations, and of the theory's use in the classroom.

It is evident that the errors in the original plots are not generally known. Certainly none of the author's dissertation committee knew, and none of them were ignorant people. The purpose of this Note is to ensure that the existence of the errors is generally known and to provide a few corrected plots to the community at large. One does not set about lightly to correct the masters, and only after numerous rederivations is there confidence that the results presented herein are correct.

### Computational Results and Discussion

The standard *V-g* method of flutter analysis for the two-degree-of-freedom (2-DOF) airfoil, Fig. 1, was implemented in MATLAB.<sup>7</sup> MATLAB's zooming feature was used to isolate the airspeed at the critical flutter point. Several plots of flutter boundaries from the literature are presented to illustrate the errors. In Figures 3, 4, and 5, BAH refers to Ref. 5, BA refers to Ref. 6, and T&G refers to Theodorsen and Garrick, either Ref. 2 or 3, specified in the text as needed.

Figure 2 shows a set of flutter boundaries vs frequency ratio for a set of values of *x*<sub>*α*</sub>. The curves in Fig. 2 were obtained from Ref. 6 (they also appear in Ref. 5). For these curves, *a* = -0.3, *μ* = 20, and *r*<sub>*α*</sub> = 0.5. For the lower values of the abscissa, there is agreement

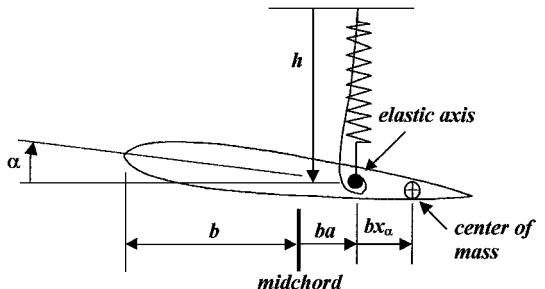


Fig. 1 Airfoil geometry, two-DOF.

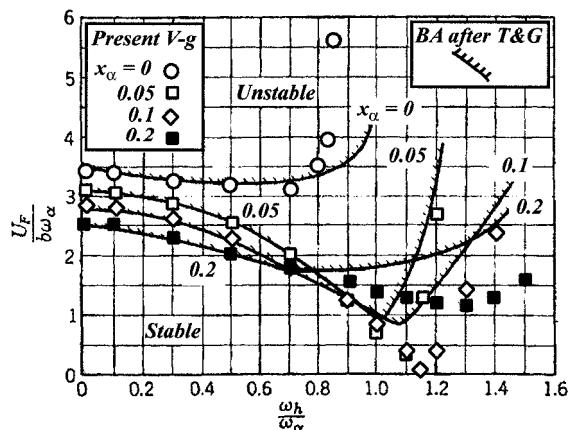


Fig. 2 First comparison of flutter boundaries from Refs. 2, 5, and 6 with present computations.

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